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(54) A LINEAR INTERPOLATING METHOD FOR COLOR SIGNALS IN A MEMORY

(71) We, DAINIPPON SCREEN SEIZO KABUSHIKI KAISHA, a Japanese Corporation, of 1-1 Tenjin-kitamashi, Teranouchi-agaru 4-chome, Horikawa-dori, Kamigyo-ku, Kyoti-shi, Japan, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to a linear interpolating method for signals in a memory which is used for color correction of picture signals in a reproducing machine such as a color scanner, a color facsimile producer, or the like, in which color separation picture images are produced by photo-electric scanning.

In conventional color photographic plate making, color correction is often made by photographic masking. However, this method has many defects, for example: limitations of color correction ability, necessity for many skilled engineers, unreliable results of the color separation, irregular quality of finish, complexity, and the like.

In order to overcome these defects, a color correction masking method by an electronic color separation machine such as a color scanner has been developed and is nowadays more popular. Most of the color scanners now used employ an analog computer system for the color correction calculations so as to increase the calculation speed.

This method, however, has also defects such as the difficulty of the introduction of many kinds of calculations because of the restriction of calculation ability, inevitable effects of temperature drift and noise, multiplicity of operational amplifiers and so forth as electric elements, inconvenience of operation due to many adjustments of potentiometers and switches, and high manufacturing cost.

If the analog computer system is simply replaced with a digital computer system, which has advantages such as a wide correction variable range and convenience of operation, the calculation speed for the color correction decreases very much, and the processing ability is reduced. Accordingly, this system is not practicable.

Recently, a direct scanner has been developed for plate making in printing, which performs color separation, color correction, conversion of scale of the reproduced image, and halftone processing at the same time so as to meet the requirement for high quality printing and rapid operation. In this case, however, there is the defect that supplementary masking or hand retouching after the color separation cannot be applied, as opposed to conventional color scanning which includes color separation, color correction, conversion of scale of the reproduction image, and halftone processing.

In general, an original color picture is scanned by a color scanner to obtain three (red, green, and blue) color separation signals. These three color separation signals are sent to a color operation circuit, thereby finally obtaining recording signals for density of printing inks, such as cyan, magenta, yellow, and black.

In order to provide the most accurate possible color reproduction, a combination of the amounts of cyan, magenta, and yellow inks (the black ink, and so forth, are omitted for the sake of brevity of explanation) is necessarily determined corresponding to a combination of red, green, and blue color separation signals.

Consequently, for the purpose of color correction by selecting the combination of cyan, magenta, and yellow values corresponding to the combination of red, green, and blue values, the color-corrected combinations of cyan, magenta, and yellow values corresponding to each combination of red, green, and blue values are stored in a memory in advance, and then the color-corrected combination of cyan, magenta, and

yellow values is read out by addressing the memory by the combination of red, green, and blue values corresponding thereto.

If each red, green, and blue range is divided into, for example, two hundred tone steps, altogether $200^3=8,000,000$ combinations of cyan, magenta, and yellow values must be stored in the memory, which requires that the memory have a large capacity. This means high cost, and thus is not practicable.

Therefore, in order to reduce the storage capacity required for the memory, each color range of red, green, and blue is divided into, for example, sixteen tone steps, and then $16^3 = 4096$ combinations of cyan, magenta, and yellow values are required. Thus the storage capacity requirement for the memory is reduced to a manageable level. On the other hand, the tone steps become too rough, and the lack of output consistency becomes conspicuous, so that printing quality suffers. Therefore, in this case, it is necessary to interpolate intermediate values properly between each two tone steps.

The present invention seeks to provide a linear interpolating method for such ssignals in a memory free from the above mentioned defects, which enables the memory to calculate quickly by using a simple formula interpolation values, without large discontinuities of the slope of the interpolated values between one interpolation unit and the next.

According to the present invention there is provided a linear interpolating method for color signals in a memory of a picture reproducing machine, comprising storing appropriate values of color picture output signals corresponding to certain stepped values of color input signals in the memory addressed in a three-dimensional fashion, and interpolating values of color output signals at points which are between said values by dividing up the cubic interpolation unit of the memory which is constituted by a single step of each of the color input signals into a plurality of tetrahedra whose vertices are either vertices of the cubic unit, centers of its faces, or its center point, calculating the color output signal at each vertex of these tetrahedra which is a center of a face of the cubic unit, if any, by averaging the values of the color output signal at the four vertices which are corners of said face, and at the center point of the cubic unit by averaging the values of the color ouput signal at all eight of the vertices of the cubic unit, determining which of these tetrahedra includes the interpolation point at which the value of the color output signal is to be interpolated, and deriving the interpolated value at the interpolation point as a weighted sum of the values at the four vertices of the determined tetrahedron, the value at each vertex being given a weight corresponding to the ratio of the volume of a second tetrahedron whose vertices are the interpolation point and the other three vertices of the determined tetrahedron to the volume of the determined tetrahedron.

In one embodiment in accordance with the invention the cubic unit is divided into twenty-four tetrahedra, of each of which one vertex is the center of the cubic unit, two vertices are vertices of the cubic unit which are connected by an edge of the cubic unit, and the fourth vertex is the center of a square face of the cubic unit one edge of which face is the said edge of the cubic unit.

In a second embodiment the cubic unit is divided into six tetrahedra by three planes which have a line in common, said line being a long diagonal of the cubic unit, and each plane containing two edges, and four vertices of the cubic unit.

In a still further embodiment the cubic unit is divided into five tetrahedra by four planes, each of which contains exactly three vertices of the cubic unit, said planes intersecting one another along lines which are diagonals of the faces of the cubic unit.

The present invention will now be described in detail with respect to the accompanying drawings, in which:

Fig. 1 is a schematic view of a conventional interpolating method over a two-dimensional interpolation unit square;

Figs. 2 and 3 are schematic views of a square interpolation unit region and a cubic interpolation unit region of the conventional two-dimensional and three-dimensional interpolating methods;

Fig. 4 is a schematic view of a distribution of interpolation values of the conventional method for two-dimensional interpolation;

Fig. 5 is a schematic view of an improved two-dimensional interpolting method;

Fig. 6 is a schematic view of a method of three-dimensional interpolation over a tetrahedral region;

Fig. 7 is a schematic view of a cubic interpolation unit dissected into twenty-four

tetrahedra according to one of the variations of the method of the present invention;

Fig. 8 is a schematic view of one of the tetrahedra of the dissection of Fig. 7;

Fig. 9 is a schematic view of another variation of the method of the present invention, wherein the unit cube is dissected into six tetrahedra;

Fig. 10 is a schematic view of the tetrahedra obtained by the dissection of Fig. 9; and

Figs. 11 and 12 illustrate another method of dissecting the unit cube into five tetrahedra, which gives another variation of the method of the present invention.

In order that the method may be better understood, some explanation of prior art methods of interpolation will now be given.

Referring to Fig. 1, there is shown an example of interpolation of a function U of two variables, where the interval to be interpolated over is taken as unity.

The value $U(x,y)$ i.e. $U(x_i+x_f,\ y_i+y_f)$ at a point P in an interpolation region ABCD will be found by a methematical interpolating method, in which x_i and y_i are the integral parts of x and y and x_f and y_f are the decimal parts.

For the interpolation it is necessary that the function at the vertices A, B, C, and D should have known values $U(x_i,y_i)$, $U(x_i+1,y_i)$, $U(x_i+1,y_i+1)$, and $U(x_i,y_i+1)$. The interpolated value $U(x,y)$ will be a function of x_f, y_f, $U(x_i,y_i)$, $U(x_i+1,y_i)$, $U(x_i+1,y_i+1)$, and $U(x_i,y_i+1)$. Further, for consistency, the interpolated value should be consistent with the known values of the original function at the corners of the unit region.

An interpolating method satisfying such a condition will be described. It is called linear interpolation because on the edges of the unit region it reduces to a simple linear interpolation function.

In order to find the value $U(x,y)$ at the point P in the interpolation unit square ABCD, first draw four perpendiculars from the point P to each side AB, BC, CD, and DA of the square. Designate the feet of these perpendiculars by Q_1, Q_2, Q_3, and Q_4 respectively, as shown in Fig. 2, and add up the results obtained by multiplying each known value at the vertices A, B, C, and D by the area of each rectangle opposite to the vertex, thereby obtaining the following equation (I):

$$\begin{aligned} U(x,y)=U(x_i+x_f,\ y_i+yf) = {} & U(x_i,y_i)\ (1-x_f)\ (1-y_f) \\ & +U(x_i+1,y_i)\ x_f\ (1-y_f) \\ & +U(x_i,y_i+1)\ (1-x_f)\ y_f \\ & +U(x_i+1,y_i+1)\ x_f\ y_f \end{aligned} \qquad (I)$$

The interpolating method according to the formula (I) satisfies the above boundary conditions at the corners of the unit square and reduces to linear interpolation along the edges of the unit square, and thus is mathematically reasonable. Further, this method may be applied to the three-dimensional case.

In Fig. 3 there is shown a unit cube interpolation unit having eight vertices with co-ordinates of

$$\begin{aligned} &(x_i,y_i,z_i),\ (x_i+1,y_i,z_i),\\ &(x_i,y_i+1,z_i),\ (x_i,y_i,z_i+1),\\ &(x_i+1,y_i+1,z_i),\ (x_i+1,y_i,z_i+1),\\ &(x_i,y_i+1,z_i+1),\ \text{and}\ (x_i+1,y_i+1,z_i+1), \end{aligned}$$

and including a point P which co-ordinates $(x_i+x_f,\ y_i+y_f,\ z_i+z_f)$ at which the value of U is to be interpolated. The cube is divided up into eight rectangular paralellopipedia by three planes which include the point P and are parallel to its faces. The value $U(x,y,z)$ at the point P is found by adding up the values obtained by multiplying each known value at each of the vertices of the unit cube by the volume of each rectangular parallelopipedon which is positioned opposite to that vertex, thereby obtaining the following formula (II):

$$
\begin{aligned}
U(x,y,z) = U(x_i+x_f,\ y_i+y_f,\ z_i+z_f) = {} & U(x_i,y_i,z_i)\ (1-x_f)\ (1-y_f)\ (1-z_f) \\
& +U(x_i+1,y_i,z_i)\ x_f\ (1-y_f)\ (1-z_f) \\
& +U(x_i,y_i+1,z_i)\ (1-x_f)\ y_f\ (1-z_f) \\
& +U(x_i,y_i,z_i+1)\ (1-x_f)\ (1-y_f)\ z_f \\
& +U(x_i,y_i+1,z_i+1)\ (1-x_f)\ y_f\ z_f \\
& +U(x_i+1,y_i,z_i+1)\ x_f\ (1-y_f)\ z_f \\
& +U(x_i+1,y_i+1,z_i)\ x_f\ y_f\ (1-z_f) \\
& +U(x_i+1,y_i+1,z_i+1)\ x_f\ y_f\ z_f
\end{aligned} \qquad \text{(II)}
$$

Again, this method produces consistent results at the vertices of the unit cube. Further, along the edges of the unit cube it reduces to simple linear interpolation, and on the faces of the unit cube it reduces to the method of quation (I). It is further clear that the value obtained in the center of each face of the unit cube is the mean value of the known values at each vertex of that face, and the value obtained at the center of the unit cube is the mean value of the eight known values at the vertices of the cube. Accordingly, this method is seen to be mathematically reasonable.

However, this method has disadvantages. It requires eight products to be formed, each of four values, and addition thereof. Hence it is not always best for high speed calculation.

There is another disadvantage in this method. Although from one unit cube to the next the interpolated values are continuous, their derivative is not. That is, the slope of the interpolated values is discontinuous from one unit cube to the next, i.e. the line of the interpolated values bends sharply as we pass over the boundary. Thus in practice a sharp step of color values will be apparent in the finished picture, and the cubic structure of the memory will show, to the detriment of quality. This effect can become quite serious. Fig. 4 shows a distribution of the interpolated values obtained according to the formula (I) which has a saddle form, which shows the aforementioned inconvenience clearly. An even continuous line of interpolated values in the unit square $A_1B_1C_1D_1$ is obtained, and also in the unit square $A_2B_2C_2D_2$, but between these two squares, at their common border, the derivative of the interpolated values is discontinuous.

The prior art interpolation methods, and their disadvantages, have been explained above. A method according to the present invention will now be described.

In Fig. 5, showing the two-dimensional case, two adjacent interpolation regions $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$ are shown. The centers of these unit squares are designated by O_1 and O_2, and interpolated values at these points are derived as averages of the function values at the four corners of the squares. Then interpolation is conducted linearly in each of the triangles $A_1O_1B_1$, $B_1O_1C_1$, $C_1O_1D_1$, $D_1O_1A_1$, $A_2O_2B_2$, $B_2O_2C_2$, $C_2O_2D_2$, and $D_2O_2A_2$. That is, the point at which the value is to be interpolated is first checked to determine which of these triangles it falls into, and then the value at the point is determined by interpolation in the triangle in a fashion analogous to Fig. 2, by drawing lines from the point to the corners of the triangle, and then calculating the value of the function at the point as a weighted sum of the values at the corners of the triangle, giving each value at a corner a weighting of the ratio of the area of a second triangle whose corners are the point and the other two corners of the triangle, and the area of the triangle. In this method the magnitude of the discontinuity in the derivative of the interpolated values from one interpolation region to the next is much reduced.

Now, considering the three-dimensional case, the basic interpolation method in a tetrahedral volume will be explained with respect to Fig. 6. Let ABCD be a tetrahedron of which each vertex is a point at which the value of the function U to be interpolated is known. The value at point P, internal to the tetrahedron, is calculated as follows: draw lines from each vertex A, B, C, and D through the point P to meet the opposite sides of the tetrahedron in A′, B′, C′, and D′. Then the interpolated value U(P) is

$$
\begin{gathered}
U(A) \times \text{ratio of volumes of tetrahedra PBCD and ABCD} \\
+ \\
U(B) \times \text{ratio of volumes of tetrahedra PDAC and ABCD} \\
+ \\
U(C) \times \text{ratio of volumes of tetrahedra PDAB and ABCD} \\
+ \\
U(D) \times \text{ratio of volumes of tetrahedra PABC and ABCD.}
\end{gathered}
$$

Now the ratio of the volumes of the tetrahedra PBCD and ABCD, for example, is the same as the ratio of the heights of P and of A from the plane of BCD.

Referring to Fig. 7, there is shown a unit cube interpolation volume ABCDEFGH, and the values of the function U of three variables are assumed to be known at the vertices of the cube. All three variations of the present method depend upon dissecting this cube into tetraheldra whose vertices are either vertices of the cube, centers of faces of the cube, or the center of the cube. Then a series of comparisons are made to determine which of these tetrahedra contains the point at which the value of the function is required to be interpolated. Once this is determined, the value is then interpolated within that tetrahedron according to the method described above, using analytical geometry. It will be realised that it is mathematically reasonable to interpolate, initially, the values of the function at centers of faces of the cube as the average of the values at the four corners of the faces, and the value of the function at the center of the cube as the average of the values at all eight vertices of the cube. Thus for each vertex of each tetrahedron of the dissection of Fig. 7 the value of the function is known, and therefore the method illustrated in Fig. 6 can be applied for interpolation.

In the dissection of Fig. 7 there exist twenty-four tetrahedra, of each of which one vertex is the center of the cubic unit, two vertices are vertices of the cubic unit which are connected by an edge of the cubic unit, and the fourth vertex is the center of a square face of the cubic unit one edge of which face is the said edge of the cubic unit. The twenty-four tetrahedra are all isomorphic. One of them is shown in Fig. 8. The centers of the faces of the cubic unit have been labelled as $Q_1 \ldots\ldots Q_6$, and the center of the cube as O. The tetrahedron $OABQ_1$ is illustrated. The planes OAB, Q_1AB, OQ_1B, and OQ_1A have the equations $y_f-z_f=0$, $z_f=0$, $x_f+y_f-1=0$, and $x_f-y_f=0$ respectively. Therefore it is clear that the condition for the point P to lie inside the tetrahedron $OABQ_1$ is that $y_f-z_f\geqslant 0$, $x_f+y_f\leqslant 0$, and $x_f-y_f\geqslant 0$. (Of course $z_f\geqslant 0$, by definition). Provided these conditions are all satisfied, the interpolated value of the function may be calculated as outlined above. Thus U(P) is equal to

$$
\begin{gathered}
U(A) \times \text{ ratio of volumes of } POQ_1B \text{ and } OABQ_1 \\
+ \\
U(B) \times \text{ ratio of volumes of } POQ_1A \text{ and } OABQ_1 \\
+ \\
U(Q_1) \times \text{ ratio of volumes of } POAB \text{ and } OABQ_1 \\
+ \\
U(O) \times \text{ ratio of volumes of } PQ_1AB \text{ and } OABQ_1 \\
= U(A)\ [1-x_f-y_f] + U(B)\ [x_f-y_f] \\
+U(Q_1)\ [2(y_f-z_f] + U(O)\ [2\ z_f] \qquad \text{(III)}
\end{gathered}
$$

This is because the ratio of the volumes of the abovementioned tetrahedra, as pointed out above, is the ratio of their heights, and the equations of their faces are as stated above.

Similar results hold when the point P is in the other interpolation tetrahedra. A complete table of the conditions for discrimination of which tetrahedron contains the point P, and of the factors which are used for calculation of the interpolated value in each case, is shown as Table 1. Using this table, by testing the conditions that are not parenthesized, it is possible to characterise the tetrahedron which contains the point P, and accordingly it is not necessary to test the parenthesized conditions.

TABLE

TETRAHEDRAL INTERPOLATION DIVISION	DISCRIMINATION CONDITIONS									
	x_f-y_f	y_f-z_f	z_f-y_f	x_f+y_f-1	y_f+z_f-1	z_f+x_f-1	[A]	[B]	[C]	[D]
ABQ_1O	+	+	(−)	−	(−)	(−)	$-(x_f+y_f-1)$	(x_f-y_f)		
BCQ_1O	+	(+)	(−)	+	(−)	−		(x_f-y_f)	(x_f+y_f-1)	
CDQ_1O	−	(+)	(−)	+	−	(−)			(x_f+y_f-1)	$-(x_f-y_f)$
DAQ_1O	−	(+)	−	−	(−)	(−)	$-(x_f+y_f-1)$			$-(x_f-y_f)$
GFQ_2O	(+)	−	−	(+)	+	(+)				
FBQ_2O	(+)	−	(−)	+	−	(+)		$-(y_f+z_f-1)$		
BCQ_2O	(+)	+	(−)	(+)	−	+		$-(y_f+z_f-1)$	(y_f-z_f)	
CGQ_2O	+	+	(−)	(+)	+	(+)			(y_f-z_f)	
GHQ_3O	−	−	(+)	+	(+)	(+)				
HEQ_3O	−	(−)	(+)	−	(+)	+				
EFQ_3O	+	(−)	(+)	−	+	(+)				
FGQ_3O	+	(−)	+	+	(+)	(+)				
ADQ_4O	(−)	+	+	(−)	−	(−)	$-(y_f+z_f-1)$			(y_f-z_f)
DHQ_4O	(−)	+	(+)	−	+	(−)				(y_f-z_f)
HEQ_4O	(−)	−	(+)	(−)	+	−				
EAQ_4O	−	−	(+)	(−)	−	(−)	$-(y_f+z_f-1)$			
AEQ_5O	+	(−)	+	(−)	(−)	−	$-(z_f+x_f-1)$			
EFQ_5O	(+)	(−)	+	(−)	−	+				
FBQ_5O	(+)	(−)	−	−	(−)	+		$-(z_f-x_f)$		
BAQ_5O	(+)	−	−	(−)	(−)	−	$-(z_f+x_f-1)$	$-(z_f-x_f)$		
CGQ_6O	−	(+)	−	(+)	(+)	+			$-(z_f-x_f)$	
CDQ_6O	(−)	(+)	−	(+)	+	−			$-(z_f-x_f)$	$-(z_f+x_f-1)$
DHQ_6O	(−)	(+)	+	+	(+)	−				$-(z_f+x_f-1)$
HGQ_6O	(−)	+	+	(+)	(+)	+				

CALCULATING FACTORS

E]	[F]	[G]	[H]	$[Q_1]$	$[Q_2]$	$[Q_3]$	$[Q_4]$	$[Q_5]$	$[Q_6]$	[O]
				$2(y_f-z_f)$						$2z_f$
				$-2(z_f+x_f-1)$						$2z_f$
				$-2(y_f+z_f-1)$						$2z_f$
				$-2(z_f-x_f)$						$2z_f$
	$-(y_f-z_f)$	(y_f+z_f-1)			$-2(z_f-x_f)$					$2(1-x_f)$
	$-(y_f-z_f)$				$2(x_f+y_f-1)$					$2(1-x_f)$
					$2(z_f+x_f-1)$					$2(1-x_f)$
		(y_f+z_f-1)			$2(x_f-y_f)$					$2(1-x_f)$
		(x_f+y_f-1)	$-(x_f-y_f)$			$-2(y_f-z_f)$				$2(1-z_f)$
$_f+y_f-1)$			$-(x_f-y_f)$			$2(z_f+x_f-1)$				$2(1-z_f)$
$_f+y_f-1)$	(x_f-y_f)					$2(y_f+z_f-1)$				$2(1-z_f)$
	(x_f-y_f)	(x_f+y_f-1)				$2(z_f-x_f)$				$2(1-z_f)$
							$2(z_f-x_f)$			$2x_f$
			(y_f+z_f-1)				$-2(x_f+y_f-1)$			$2x_f$
$_f-z_f)$			(y_f+z_f-1)				$-2(z_f+x_f-1)$			$2x_f$
$_f-z_f)$							$-2(x_f-y_f)$			$2x_f$
$-x_f)$								$2(x_f-y_f)$		$2y_f$
$-x_f)$	(z_f+x_f-1)							$-2(y_f+z_f-1)$		$2y_f$
	(z_f+y_f-1)							$-2(x_f+y_f-1)$		$2y_f$
								$-2(y_f-z_f)$		$2y_f$
		(z_f+x_f-1)							$-2(x_f-y_f)$	$2(1-y_f)$
									$2(y_f+z_f-1)$	$2(1-y_f)$
			(z_f-x_f)						$2(x_f+y_f-1)$	$2(1-y_f)$
		(z_f+x_f-1)	(z_f-x_f)						$2(y_f-z_f)$	$2(1-y_f)$

It is readily understood that the calculation is far simpler in practice than the method of the abovementioned formula (II). Further, in this method, the discontinuities across the borders between one unit cube and the next are much reduced, since the values near the face of the unit cube are much more dominated by the values at the four corners of the face than in the prior art method of (II).

In fact the color picture output signals in the memory commonly vary monotonically, and therefore a more simple and coarse method of interpolation than the one outlined above may well be satisfactory in a particular case. Therefore the method of Figs. 9 and 10 may well be acceptable, although it is not quite so accurate as the method of formula (III). In Fig. 9 is shown a dissection of the unit cube into tetrahedra all of those vertices are vertices of the unit cube. Thus this method has the advantage that no averaging of values at the vertices of the unit cube is necessary in order to determine values at the centers of the faces of the unit cube and at its center.

The unit cube is dissected into six tetrahedra by three planes which have a line in common which is the long diagonal of the unit cube, and each plane is inclined to the other two at 60° and contains two edges and four vertices of the unit cube. A typical one of the six tetrahedra is illustrated in Fig. 10. In this case the conditions for the point P to lie within this tetrahedron are that $x_t \geqslant y_t \geqslant z_t$, as can be easily worked out using solid geometry, as before. In the same way the interpolated value U(P) is equal to

$$U(A)\ [1-x_t) + U(B)\ [x_t-y_t] + U(C)\ [y_t-z_t] + U(D)\ z_t.$$

Similar discriminating conditions and calculating factors can be worked out for the other five tetrahedra. Table 2 shows the complete set. It is readily appreciated that this variation of the method is easier in calculation than the method of formula (III), albeit at a slight loss in accuracy.

TABLE 2

DISCRIMINATION CONDITIONS ↓	CALCULATING FACTORS							
	$u(x_i.y_i.z_i)$	$u(x_i+1.y_i.z_i)$	$u(x_i.y_i+1.z_i)$	$u(x_i.y_i.z_i+1)$	$u(x_i+1.y_i+1.z_i)$	$u(x_i+1.u_i.z_i+1)$	$u(x_i.y_i+1.z_i+1)$	$u(x_i+1.y_i+1.z_i+1)$
$x_f \geqq y_f \geqq z_f$	$1-x_f$	x_f-y_f			y_f-z_f			z_f
$x_f \geqq z_f > y_f$	$1-x_f$	x_f-z_f				z_f-y_f		y_f
$z_f > x_f \geqq y_f$	$1-z_f$		z_f-x_f			x_f-y_f		y_f
$z_f \geqq y_f > x_f$	$1-z_f$		z_f-y_f				y_f-x_f	x_f
$y_f > z_f \geqq x_f$	$1-y_f$			y_f-z_f			z_f-x_f	x_f
$y_f > x_f > z_f$	$1-y_f$			y_f-x_f	x_f-z_f			z_f

In Figs. 11 and 12 there is shown another method for dissecting the unit cube into tetrahedra, which this time are five in number. The unit cube is divided up by four planes, each of which contains exactly three vertices of the cube, and which are characterized by intersecting one another along lines which are diagonals of faces of the cube. Thus there are two possible dissections which are mirror images of one another, and these are shown in the figures. Exploded diagrams also show how the tetrahedra fit together. In this variation it will be noted that the tetrahedra are not all isomorphic; one is different from the others. As before, it is determined using discrimination conditions derived from solid geometry in which of these tetrahedra the interpolation point lies, and then, using calculation factors derived in the same way as above, the interpolated value is calculated. It will be obvious to anyone skilled in the art, depending upon the foregoing disclosure, how to calculate these discrimination conditions and calculation factors, and therefore listing of them will be omitted for the sake of brevity of explanation.

In the methods which use the dissections into six and into five tetrahedra, which are explained above with reference to figures 9, 10, 11, and 12, in fact the interpolated values at the center of the faces of the unit cube, and at its center, will be different slightly from those derived by simple averaging which were used in the first version

of the method, illustrated in Fig. 7. However, this variation will only be slight in the case of monotonic functions, and is quite tolerable.

WHAT WE CLAIM IS:—

1. A linear interpolating method for color signals in a memory of a picture reproducing machine, comprising storing appropriate values of color picture output signals corresponding to certain stepped values of color input signals in the memory addressed in a three-dimensional fashion, and interpolating values of color output signals at points which are between said values by:

dividing up the cubic interpolation unit of the memory which is constituted by a single step of each of the color input signals into a plurality of tetrahedra whose vertices are either vertices of the cubic unit, centers of its faces, or its center point;

calculating the color output signal at each vertex of these tetrahedra which is a center of a face of the cubic unit, if any, by averaging the values of the color output signal at the four vertices which are corners of said face, and at the center point of the cubic unit by averaging the values of the color output signal at all eight of the vertices of the cubic unit;

determining which of these tetrahedra includes the interpolation point at which the value of the color output signal is to be interpolated; and

deriving the interpolated value at the interpolation point as a weighted sum of the values at the four vertices of the determined tetrahedron, the value at each vertex being given a weight corresponding to the ratio of the volume of a second tetrahedron whose vertices are the interpolation point and the other three vertices of the determined tetrahedron to that of the determined tetrahedron.

2. A method as in Claim 1, wherein the cubic unit is divided into twenty-four tetrahedra, of each of which one vertex is the center of the cubic unit, two vertices are vertices of the cubic unit which are connected by an edge of the cubic unit, and the fourth vertex is the center of a square face of the cubic unit one edge of which face is the said edge of the cubic unit.

3. A method as in Claim 1, wherein the cubic unit is divided into six tetrahedra by three planes which have a line in common, said line being a long diagonal of the cubic unit, and each plane containing two edges and four vertices of the cubic unit.

4. A method as in Claim 1, wherein the cubic unit is divided into five tetrahedra by four planes, each of which contains exactly three vertices of the cubic unit, said planes intersecting one another along lines which are diagonals of faces of the cubic unit.

5. A linear interpolating method for colour signals in a memory of a picture reproducing machine as claimed in Claim 1 and substantially as hereinbefore described with reference to and as illustrated in Figs. 5, 6, 7 and 8, 9 and 10 or 11 and 12 of the accompanying drawings.

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FIG. 1

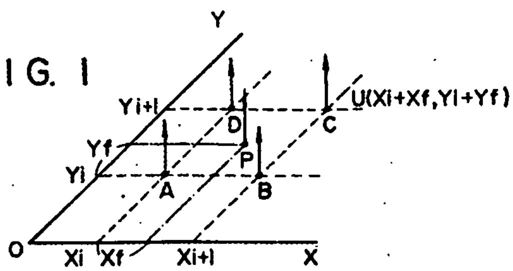


FIG. 2

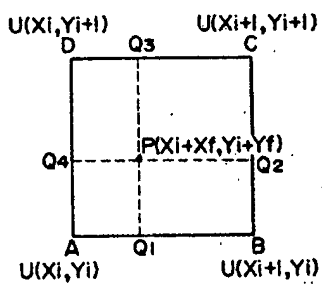


FIG. 3

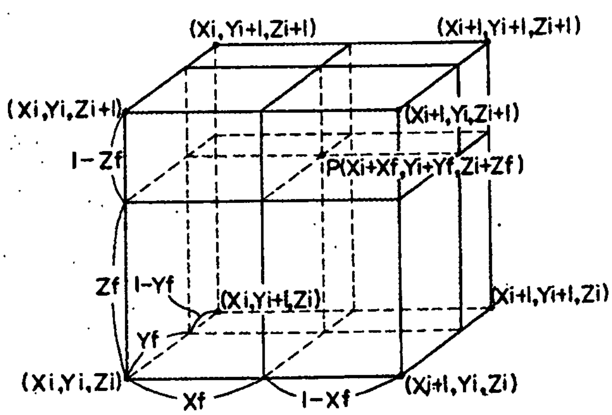


FIG. 4

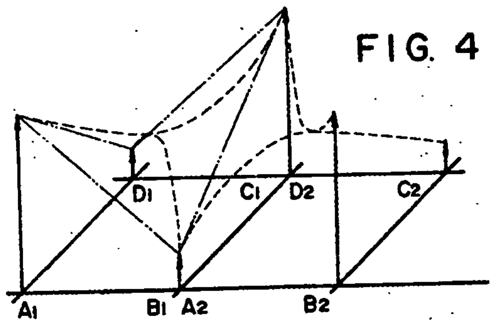


FIG. 5

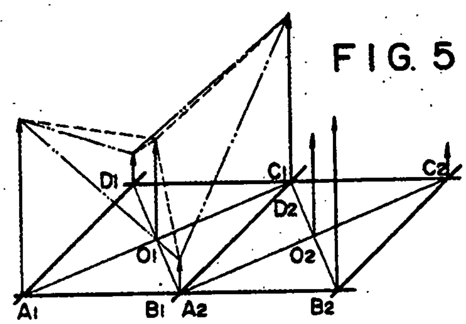
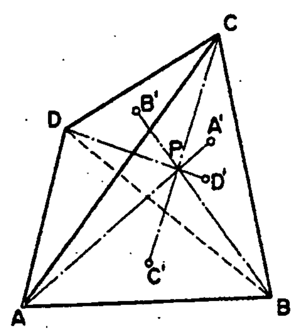


FIG. 6



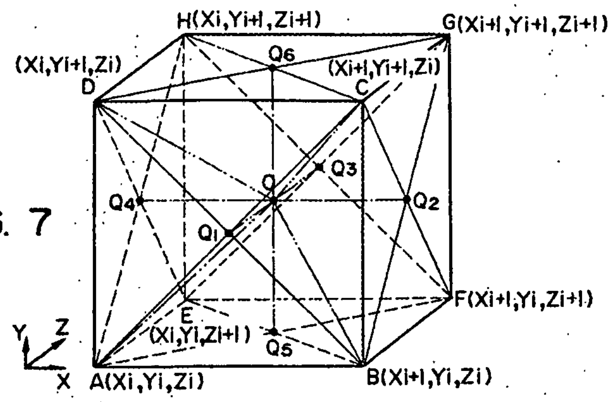


FIG. 7

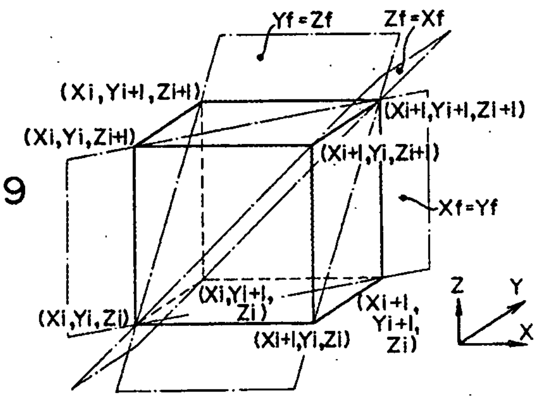


FIG. 9

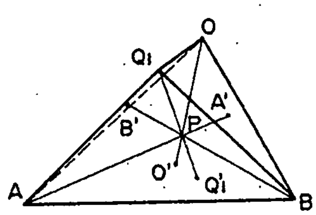


FIG. 8

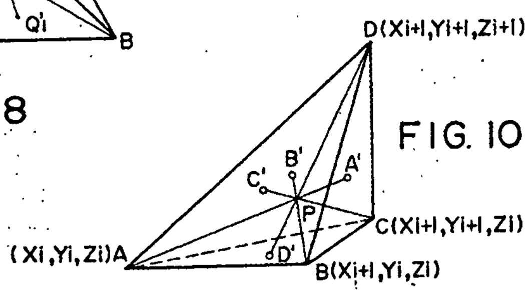


FIG. 10

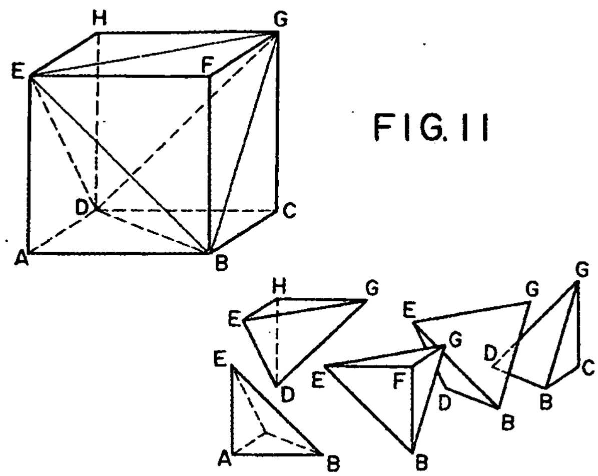

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FIG. 11

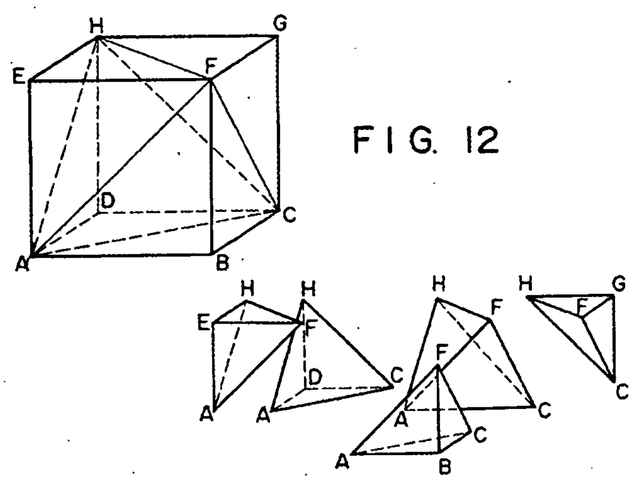
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FIG. 12